Motion under constant acceleration

Stage 1 physics

Equations work with

- Displacement (not distance) $\vec{s}$
- Velocity (not speed) $\vec{v}$
- Acceleration $\vec{a}$

The ‘arrow’

$$\vec{s}$$

The arrow denotes that this is a vector quantity

Vectors have both magnitude and direction

$\vec{s}$ is displacement; the straight line distance between two points

It has a magnitude (the length)

And a direction

Scalars vs Vectors

Scalar	Vector
Distance	Displacement
Speed	Velocity

Speed (scalar)

- Speed vs Average Speed
  - At any point as you drive to Melbourne, your speed (according to your speedo) may reach 110kph
  - However, when you get to Melbourne and calculate your average speed (based on total distance covered divided by total time taken), you may find the average speed is only 90kph
  - Note; speed and average speed are scalars and therefore do not have a direction

Velocity (vector)

- Velocity vs Instantaneous Velocity
  - At any point as you drive to Melbourne, your Instantaneous Velocity (according to your speedo AND a compass) may be 30.6ms^{-1} to the East
  - However, when you get to Melbourne and calculate your velocity (based on the displacement of Melbourne From Adelaide divided by change in time from start to finish), you may find the velocity is 25ms^{-1}
  - Note; we use quite different terminology between speed and velocity.

The starting point

The definition of average velocity

$$\vec{v} = \frac{\Delta s}{\Delta t} = \frac{v_0 + v}{2}$$

The definition of average acceleration

$$\vec{a} = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{\Delta t}$$

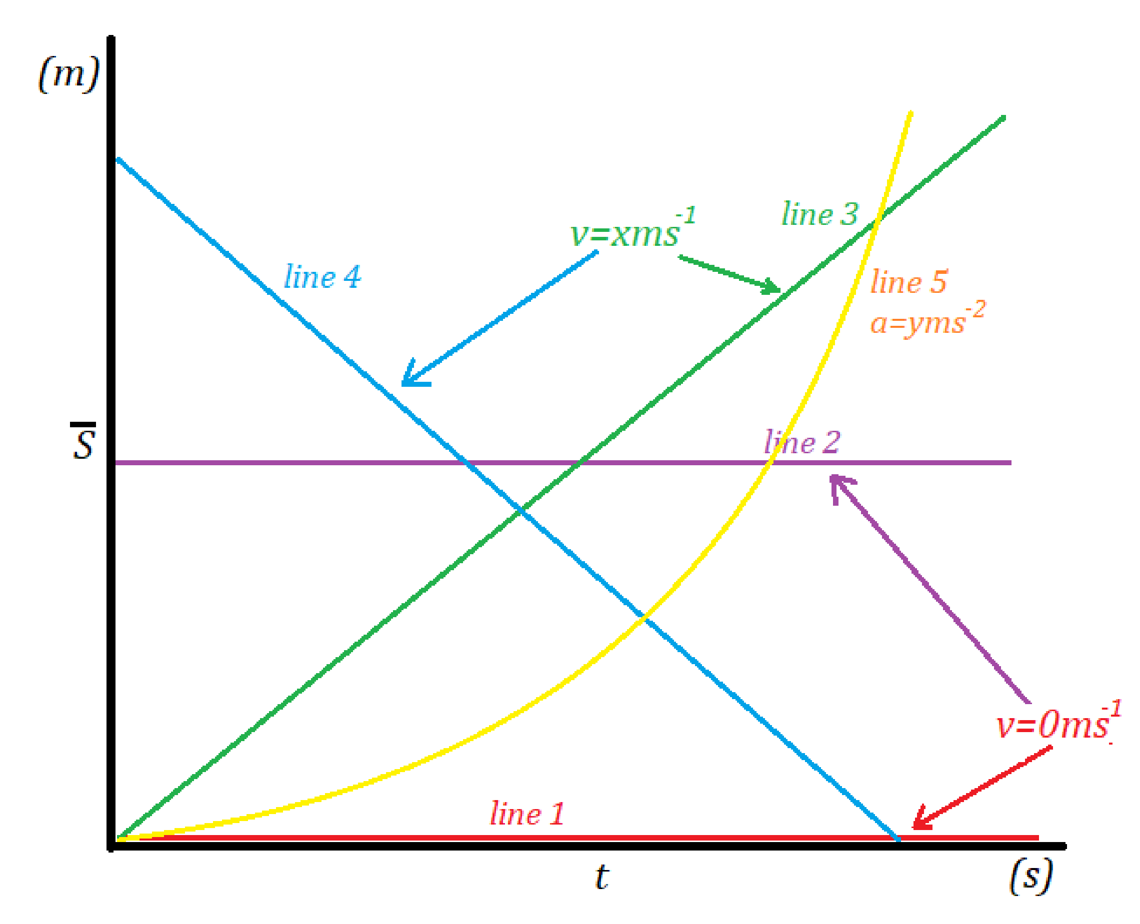
Always use SI units

- Displacement in metres (m)
- Time in seconds (s)
- Thus
  - Velocity is in ms^{-1}
  - Acceleration is in ms^{-2}

- If not, convert

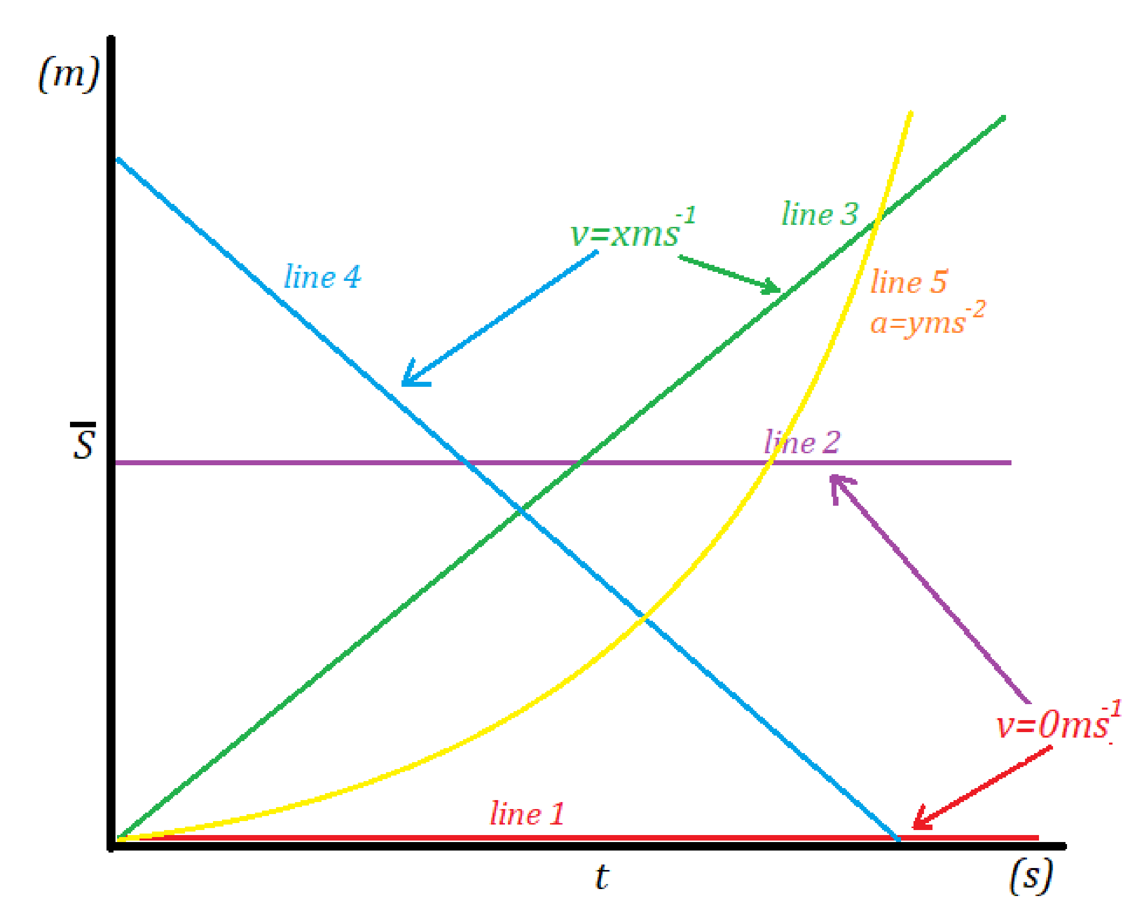
Graphic representations (s/t graph)

- Line 1
  - It is at the starting point ($\vec{s} = 0m$)
  - Displacement does not change
  - Velocity $= 0ms^{-1}$
  - Acceleration $= 0ms^{-2}$



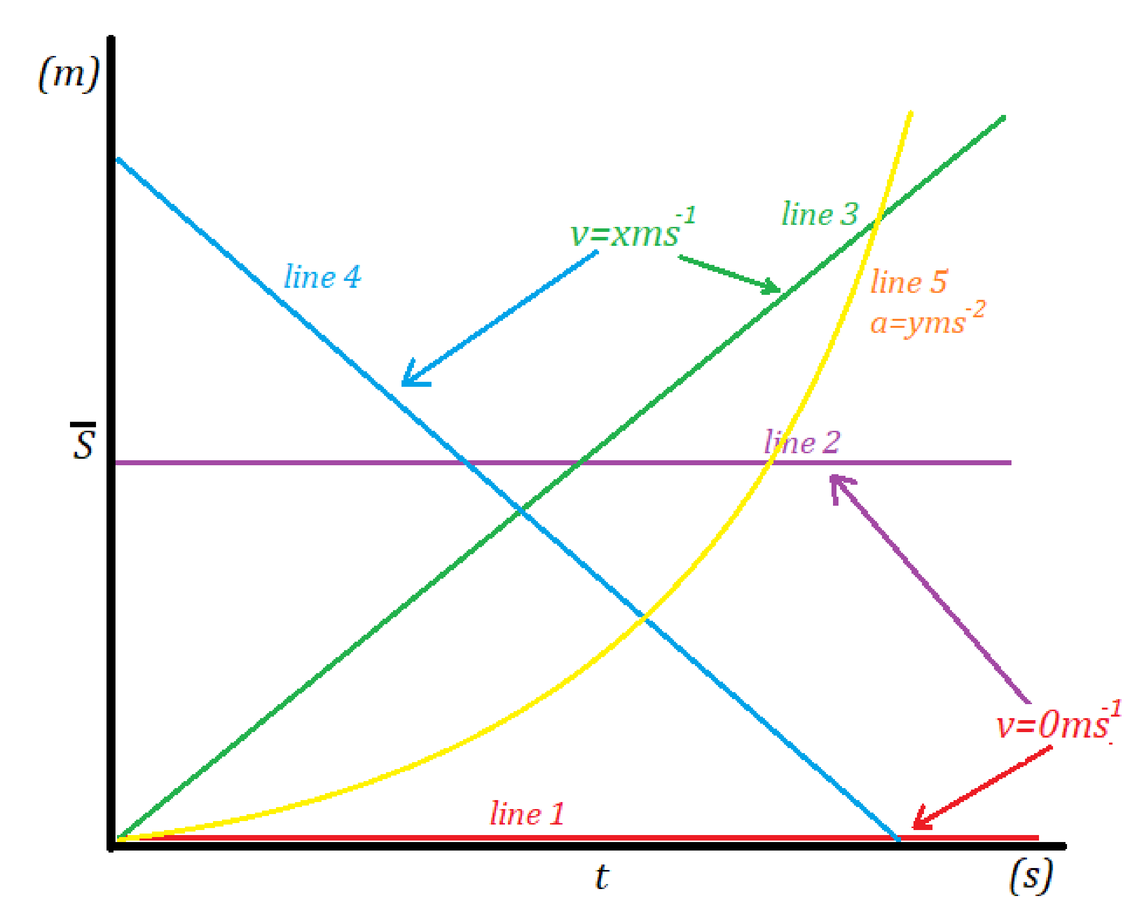
Graphic representations (s/t graph)

- Line 2
  - It is at a fixed distance from the starting point
  - Displacement does not change
  - Velocity $= 0ms^{-1}$
  - Acceleration $= 0ms^{-2}$



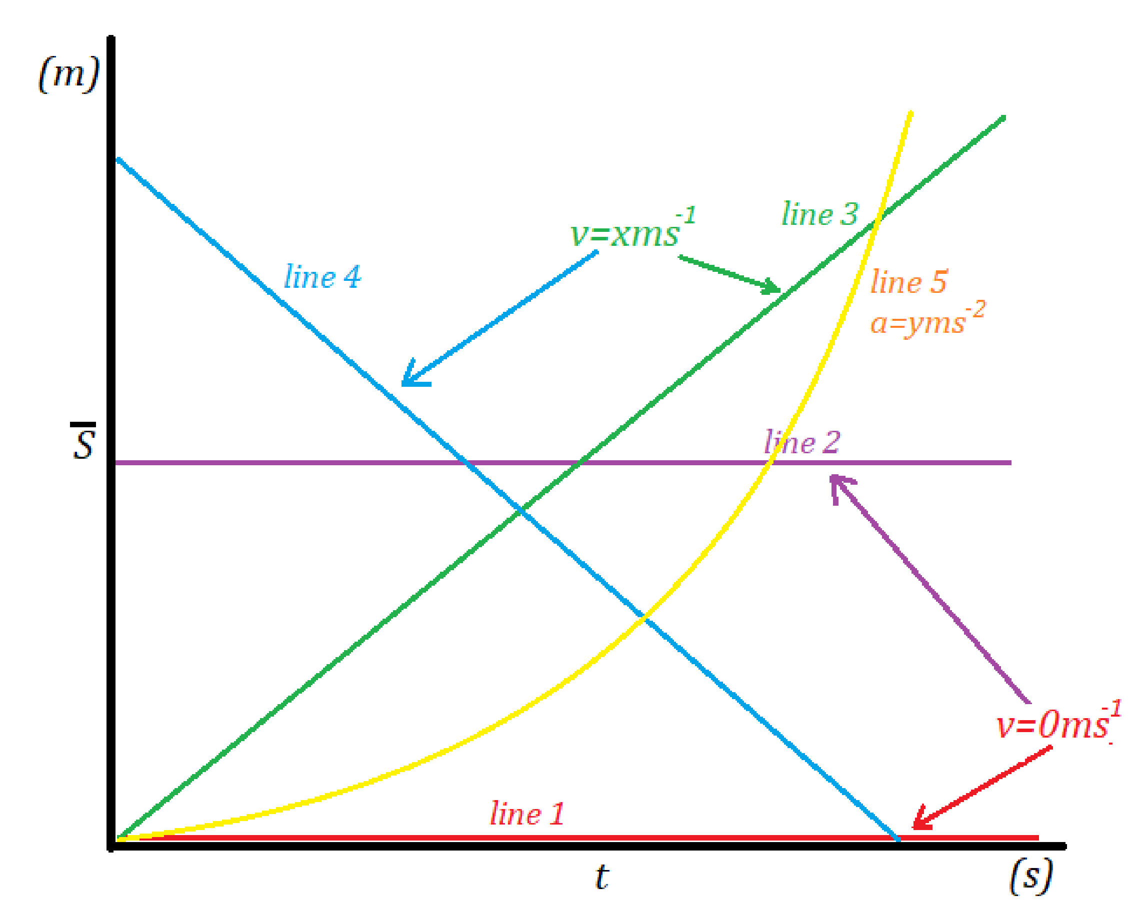
Graphic representations (s/t graph)

- Line 3
  - It is moving at a constant rate away from the starting point
  - Displacement is constantly changing
  - Velocity $= xms^{-1}$
  - Acceleration $= 0ms^{-2}$



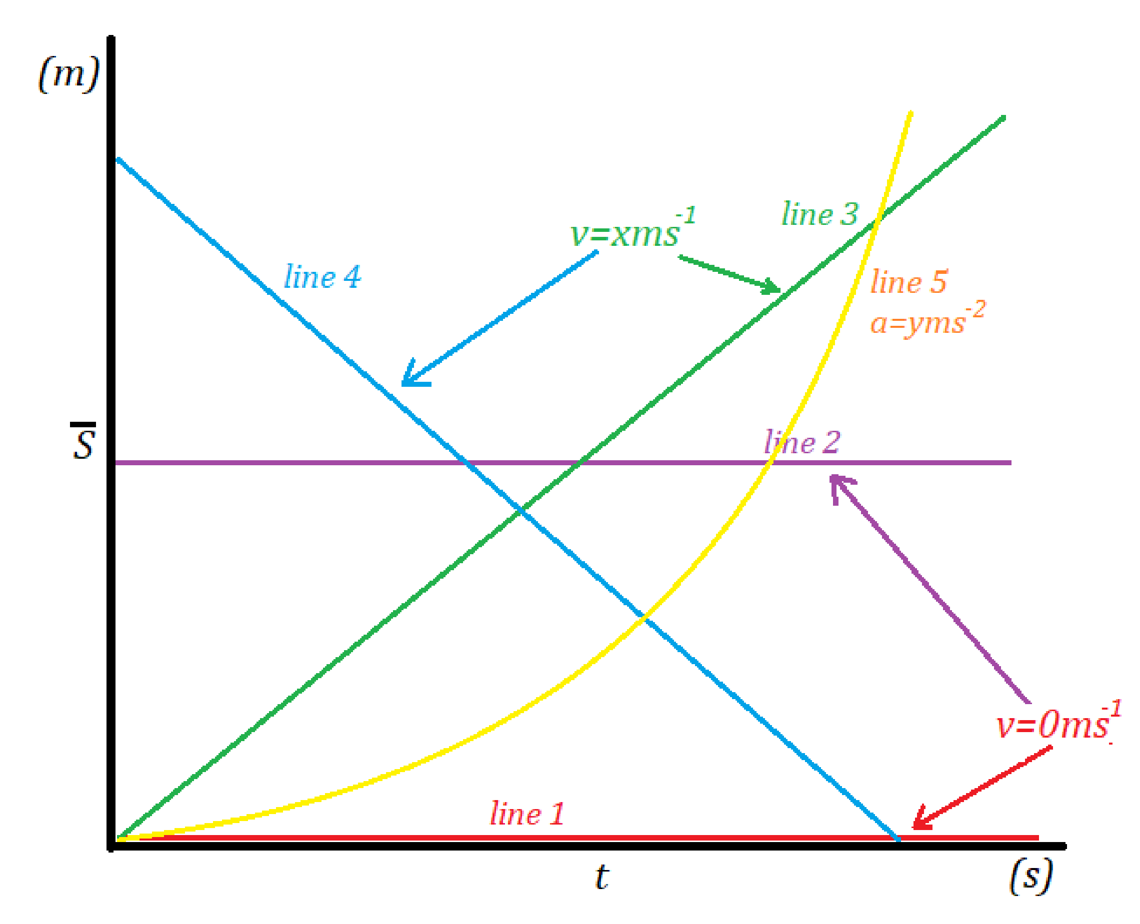
Graphic representations (s/t graph)

- Line 4
  - It is moving at a constant rate toward the starting point
  - Displacement is constantly changing
  - Velocity $= -xms^{-1}$
  - Acceleration $= 0ms^{-2}$



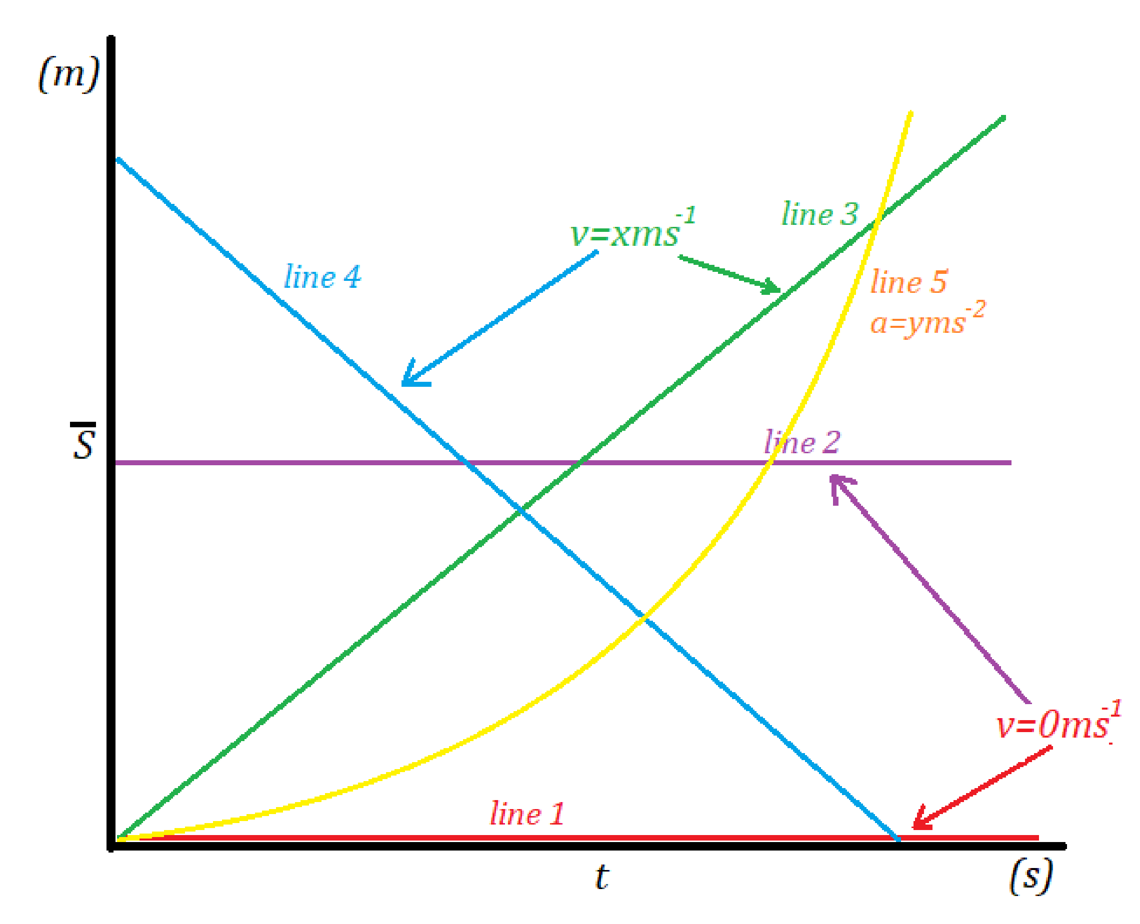
Graphic representations (s/t graph)

- Line 5
  - It is moving at an increasing rate away from the starting point
  - Displacement is constantly changing
  - Velocity is constantly increasing
  - Acceleration $= yms^{-2}$

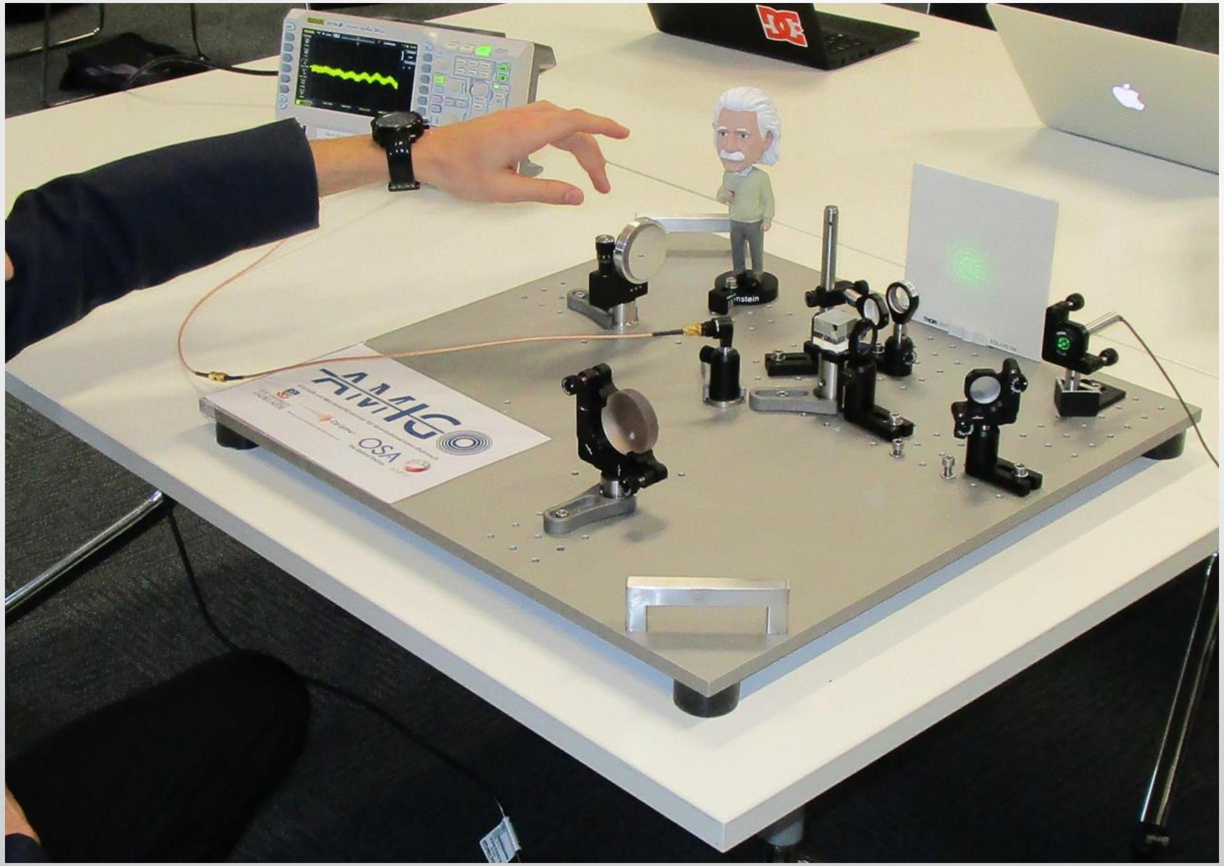


The s/t graph

- An s/t graph shows the displacement from a point
- The gradient of a line is its velocity
- Straight lines denote constant velocity
  - Horizontal – velocity is $0ms^{-1}$
- Curved lines denote changing velocity – acceleration $\neq 0ms^{-2}$



Break

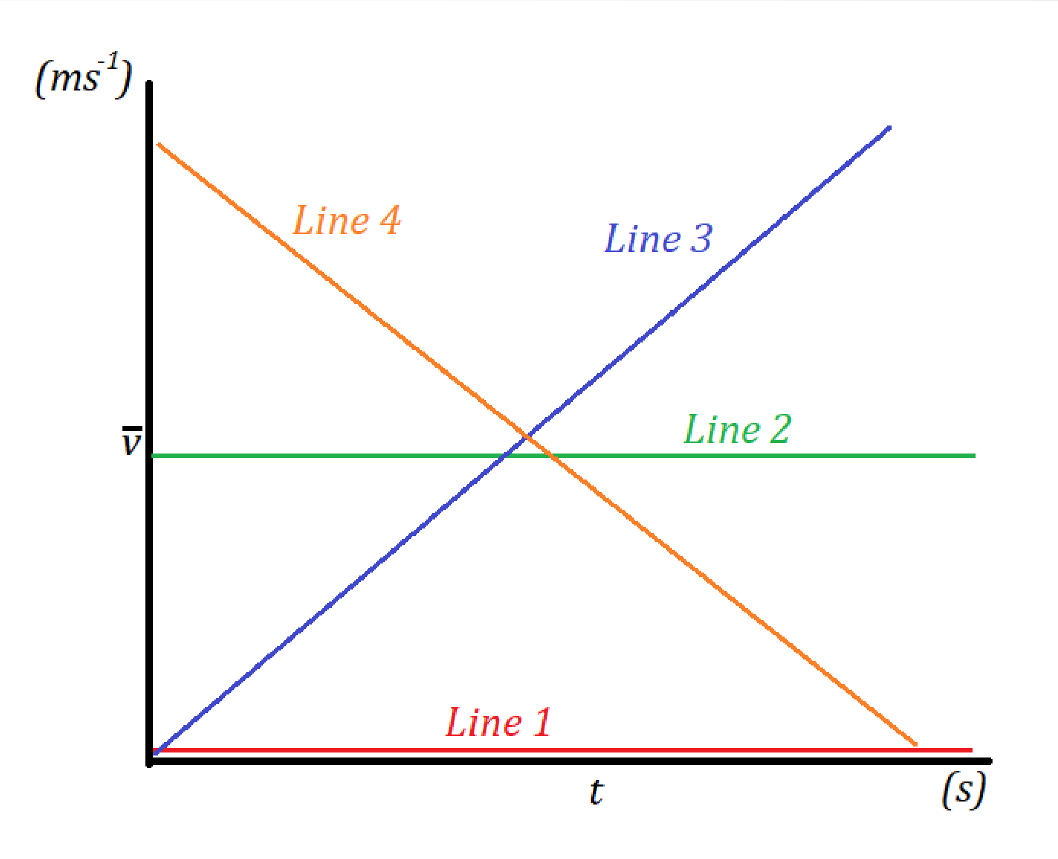


The velocity / time graph

- Note; This is a new graph. We are now plotting velocity verses time for an object.

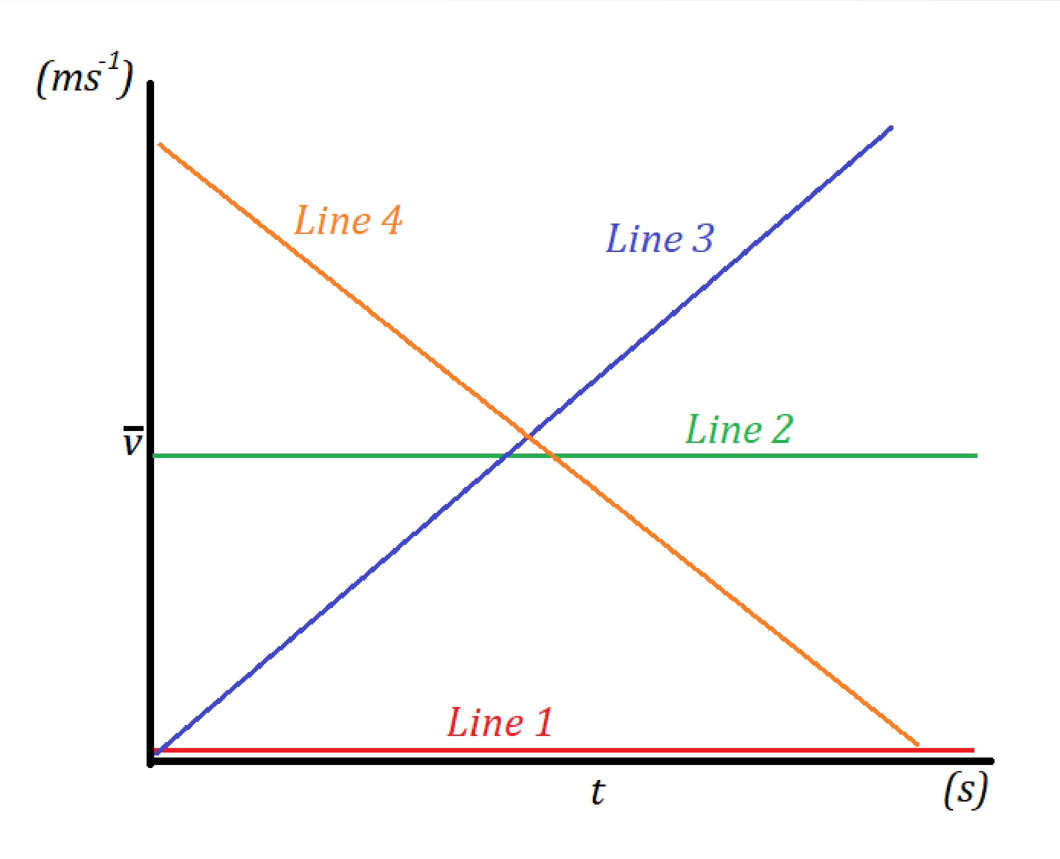
The v/t graph

- Line 1
  - Velocity is zero
  - Acceleration is zero
  - Note; this does not tell you "where" the object is, only that it is currently stationary



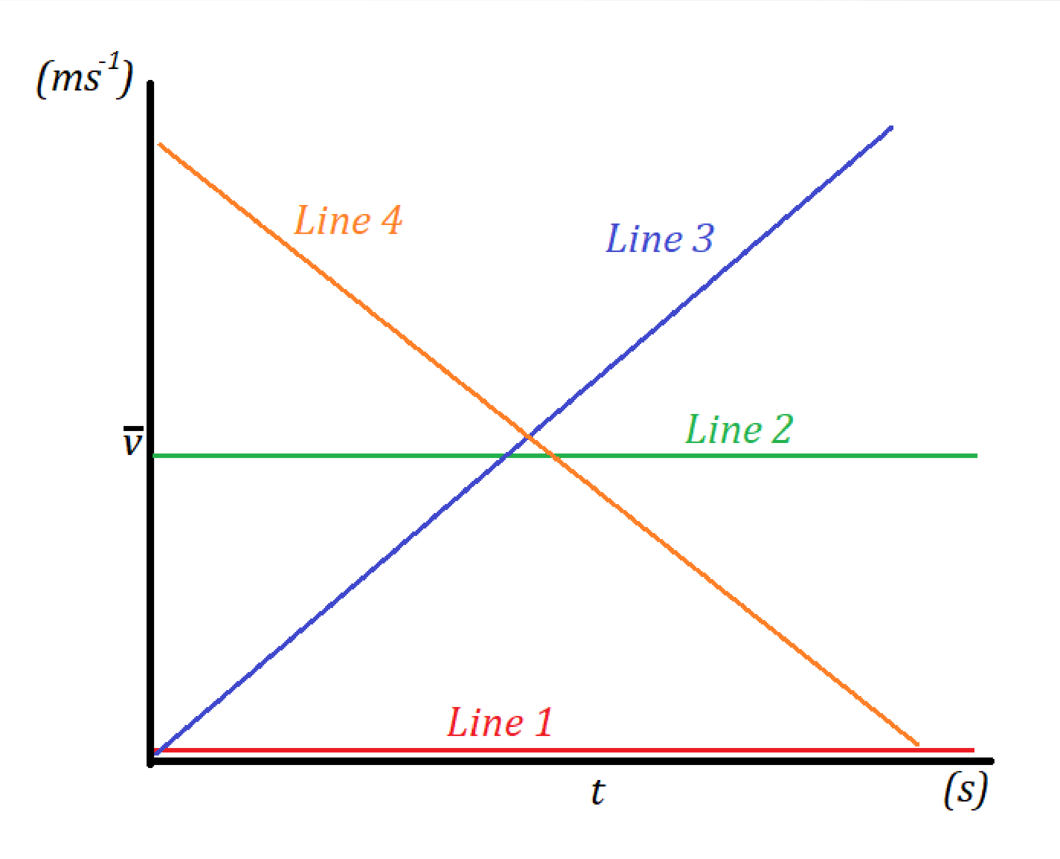
The v/t graph

- Line 2
  - Velocity is constant and positive
  - Acceleration is zero
  - Note; Again this does not tell you “where” the object is, only that it is currently moving with a constant velocity



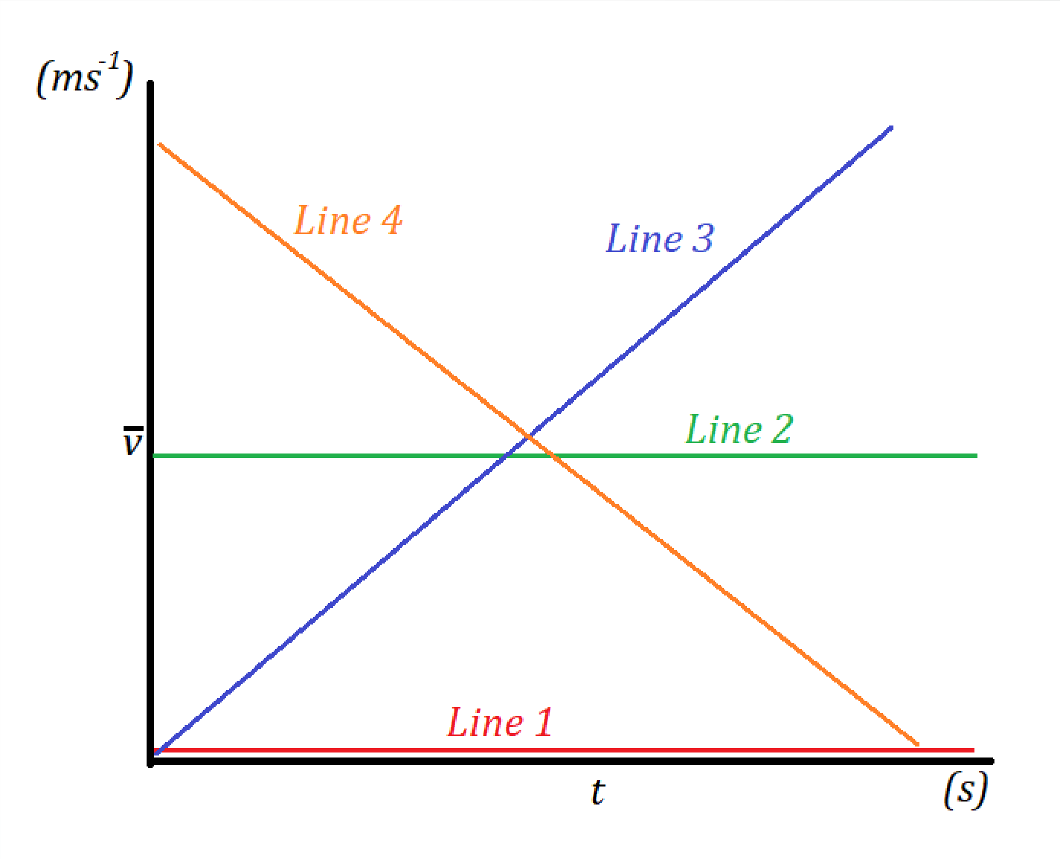
The v/t graph

- Line 3
  - The velocity is steadily increasing from rest
  - Acceleration is positive and constant
  - Note; Again this does not tell you “where” the object is, only that it is currently moving with an increasing velocity



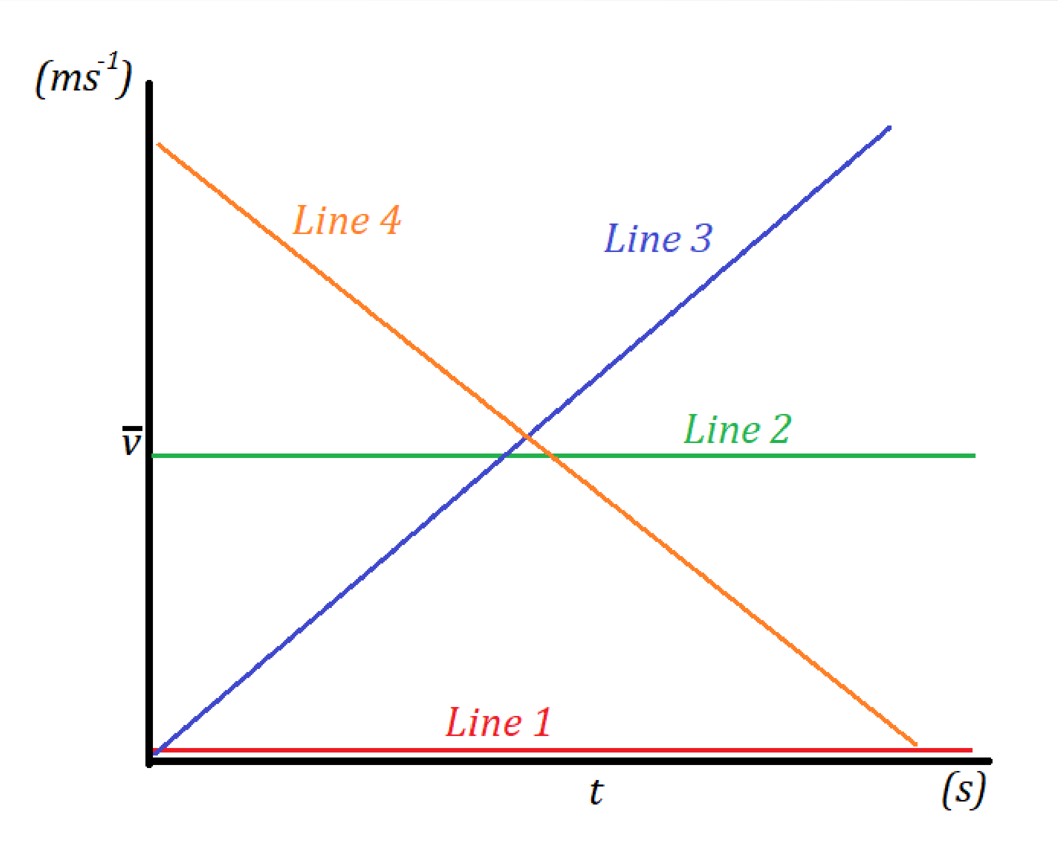
The v/t graph

- Line 4
  - The velocity is steadily decreasing to rest
  - Acceleration is negative and constant
  - Note; Again this does not tell you "where" the object is, only that it is currently moving with an decreasing velocity



The v/t graph

- A v/t graph shows the velocity at a time
- The gradient of a line is its acceleration
- Horizontal lines denote constant velocity; $a = 0ms^{-2}$
- Sloping lines denote changing velocity – acceleration $\neq 0ms^{-2}$
- The area under the v/t graph is the displacement



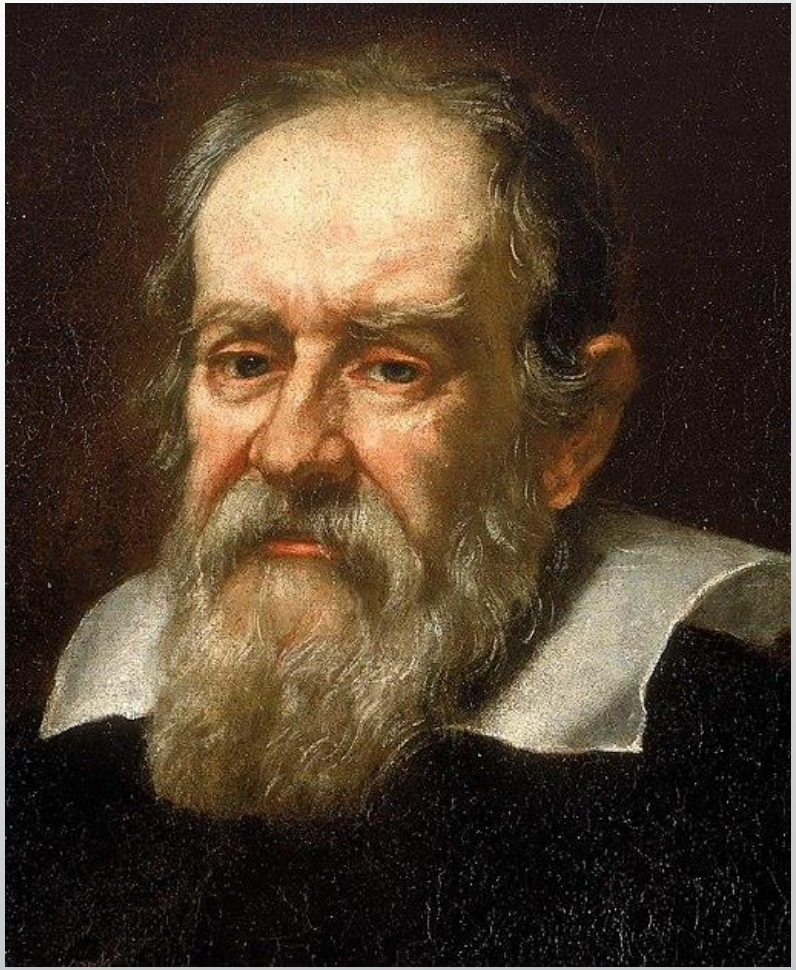
Recap on slope

- The slope of the s/t graph is velocity
- The slope of the v/t graph is acceleration
- And the area under the v/t graph is displacement
- And ….the area under an a/t graph is the velocity change in that time period

Time for another break

Galileo

- Though working before Newton was born, Galileo Galilei was one of the great contributors to our understanding of classical physics and astronomy. His work also underpins much of Einstein's theories of relativity.
- He is also attributed with the 3 main equations of motion for bodies with uniform acceleration.



Equation 1

$$\vec{v} = \vec{v}_0 + \vec{a}t$$

$\vec{v}$ $(no\ subscript\ is\ always\ final\ velocity)$

$\vec{v}_0$ $is\ initial\ velocity\ (velocity\ at\ time\ t = 0)$

$\vec{a}$ $is\ acceleration$

$t\ is\ the\ time\ from\ start\ to\ finish\ (also\ \Delta t)$

Example without re-arrangement

- John's dog is running at $3ms^{-1}$ when it accelerates after a cat at $2ms^{-2}$ for 3 seconds.
- What is its final velocity?

Solution structure

- Step 1 – declare all variables
  - John's dog is running at $3ms^{-1}$ when it accelerates after a cat at $2ms^{-2}$ for 3 seconds

  So

  - $\vec{v}_0 = 3ms^{-1}$
  - $\vec{a} = 2ms^{-2}$
  - $t = 3s$

Solution structure 2

- 2a - Declare the definition equation

$$\vec{v} = \vec{v}_0 + \vec{a}t$$

- 2b – Manipulate the equation if required (algebraic manipulation)
  - Not required for this example

Solution structure 3

- 3 – Substitute values for the variables and solve
- $\vec{v} = 3 + 2 \times 3$
- $\vec{v} = 9$
- *4 – Declare final answer to correct significant figures with units and scientific notation (if required), double underline your final answer*
- $\vec{v} = 9ms^{-1}$

Solution without notation

- John's dog is running at 3ms^{-1} when it accelerates after a cat at 2ms^{-2} for 3 seconds. What is its final velocity?
  - $\vec{v}_0 = 3ms^{-1}$
  - $\vec{a} = 2ms^{-2}$
  - $t = 3s$
- $\vec{v} = \vec{v}_0 + \vec{a}t$
- $\vec{v} = 3 + 2 \times 3$
- $\vec{v} = 9$
- $\vec{v} = 9ms^{-1}$

Example with re-arrangement

- A shopping trolley gets away from a customer, rolling down a hill and hitting a wall at $12ms^{-1}$. If it rolls for 3 seconds before hitting the wall and was going at $2ms^{-1}$. what was its acceleration?

The solution

- $\vec{v}_0 = 2ms^{-1}$
- $\vec{v} = 12ms^{-1}$
- $t = 3s$

- $\vec{v} = \vec{v}_0 + \vec{a}t$
- *New step – manipulate or re-arrange the equation (do the algebra)*
- $\vec{a}t = \vec{v} - \vec{v}_0$
- $\therefore \vec{a} = \frac{\vec{v}-\vec{v}_0}{t}$
- $\therefore \vec{a} = \frac{12-2}{3}$
- $\vec{a} = 3.33$
- $\vec{a} = 3ms^{-2}$
- Note the significant figures; since the question gives only a certainty of 1 sig fig, so should the final answer

Significant figures note

- Always give final answer to the least number of significant figures given in the question
- Always give second last line of the solution to sig figs + 2
- Always calculate to sig figs + 2

Equation 2

$$\vec{s} = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$$

$\vec{s}$ *is the displacement; the straight line distance between the starting and finishing points.*

Example without rearrangement

- What is the displacement of a car over 5 seconds if it was going at $3ms^{-1}$ initially and then accelerated for the 5 seconds at $3ms^{-2}$?

Example with rearrangement

- How long has a ball been falling if it was dropped and when observed by a teacher who is 4 stories below the student who dropped it? Each story is 3m high.

Equation 3

$$\vec{v}^2 = \vec{v}_0^{\,2} + 2\vec{a}\vec{s}$$

Example without rearrangement

- What is the velocity of a car that has accelerated for 20m at $3ms^{-2}$, having been rolling at $4ms^{-1}$ at the time you first observed it?

Example with re-arrangement

- What distance has a rock fallen if when you see it, it is falling at 290ms^{-1}

The 3 Equations of Motion

$$\vec{v} = \vec{v}_0 + \vec{a}t$$

$$\vec{s} = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$$

$$\vec{v}^2 = \vec{v}_0{}^2 + 2\vec{a}\vec{s}$$